

PA- MDC Grade 4

Representing Multi-Digit Multiplication Multiple Ways

***Concept
Development
Formative
Assessment
Lesson***

This lesson is adapted from the Kentucky Department of Education. Materials were adapted from *Everyday Mathematics*, *Uncovering Student Misconceptions in Mathematics*, and the National Library of Virtual Manipulatives. Teacher notes taken from *Teaching Student-Centered Mathematics*.

This Formative Assessment Lesson is designed to be part of an instructional unit. This task should be implemented approximately two-thirds of the way through the instructional unit. The results of this task should be used to inform the instruction that will take place for the remainder of your unit.

Mathematical goals

This concept-based lesson is intended to help you assess how well students are able to use a variety of strategies to multiply. In particular, this unit aims to identify and help students who have difficulties with:

- Traditional multiplication algorithm.
- Representing multiplication in multiple ways.

Common Core Standards

This lesson involves a range of *mathematical practices* from the standards, with emphasis on:

2. Reason abstractly and quantitatively.
7. Look for and make use of structure.
8. Look for and make use of repeated reasoning.

This lesson asks students to select and apply mathematical content from across the grades, including the *content standards*:

Number and Operations in Base Ten

4-NBT: Use place value understanding and properties of operations to perform multi-digit arithmetic.

5-NBT: Perform operations with multi-digit whole numbers.

PA Core Standards

CC.2.1.4.B.1 Apply place value concepts to show an understanding of multi-digit whole numbers.

CC.2.1.4.B.2 Use place value understanding and properties of operations to perform multi-digit arithmetic.

Introduction

This lesson is structured in the following way:

- Before the lesson, students work individually on a pre-assessment that is designed to reveal their current understanding and difficulties. You then review their work, and formulate questions for students to answer, to help them improve their solutions.
- During the lesson, students work in small groups of two or three to match the word problem, model, and multiple strategies of the same multiplication or division problem.
- In a whole-class discussion, explain their answers.

- Finally, students return to their original assessment task, to improve their responses.

Materials required

Each individual student will need two copies of the worksheet: *Representing Multi-Digit Multiplication Multiple Ways*

- Each small group of students will need a packet of Card Set A - F copied in color cut up before the lesson. {Note: you may want to make color copies, and laminate these for use in multiple classes over multiple years.}

Time needed: 90 minutes

- Approximately 15 minutes for the pre-assessment
- 60 minute lesson
- 15 minutes for the students to review their work for changes.

All timings are approximate. Exact timings will depend on the needs of the class.

Before the lesson

Assessment task: Have the students do the pre-assessment in class a few days before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. Then you will be able to target your help more effectively in the follow-up lesson.

Give each student a copy *Representing Multi-Digit Multiplication Multiple Ways*. Introduce the pre-assessment briefly and help the class to understand the problem and context.

Frame the task:

Spend the next fifteen minutes, on your own, answering the three questions to the best of your ability.

Don't worry if you do not complete or fully understand all that is being asked of you.

We will have a follow-up lesson to assist you in understanding.

It is important that students answer the question without any assistance, as far as possible. If students are struggling to get started, ask them questions that help them understand what is required.

Mathematics Formative Assessment Lesson Student Materials Revised Alpha Version

Multiplication Strategies & Representations Task

1.) Multiply 28 by 17 and show your work:

2.) Sam, Julie, Pete, Lisa, & Fred each multiplied 28 by 17. Below each method indicate if the work is correct and then explain whether that method makes sense mathematically or not.

Sam	Julie	Pete	Lisa	Fred
$\begin{array}{r} 28 \\ 17 \\ \hline 196 \\ + 280 \\ \hline 476 \end{array}$		$\begin{aligned} 28 \times 10 &= 280 \\ 28 \times 5 &= 140 \\ 28 \times 2 &= 56 \\ 280 + 140 + 56 &= 476 \end{aligned}$	$\begin{array}{r} 20 \quad 8 \\ 10 \quad 10 \quad 10 \\ 10 \quad 10 \quad 10 \\ \hline 200 \quad 140 \quad 80 \quad 56 \\ \hline 476 \end{array}$	
Check one: <input type="checkbox"/> correct <input type="checkbox"/> incorrect Explain why:	Check one: <input type="checkbox"/> correct <input type="checkbox"/> incorrect Explain why:	Check one: <input type="checkbox"/> correct <input type="checkbox"/> incorrect Explain why:	Check one: <input type="checkbox"/> correct <input type="checkbox"/> incorrect Explain why:	Check one: <input type="checkbox"/> correct <input type="checkbox"/> incorrect Explain why:

3.) Which method most closely matches how you solved the original problem? Choose a different method than what you used in #1 to multiply 35 by 14. Show your work below:

Assessing Students' Responses

We suggest that you do not score students' work. The research shows that this is counterproductive, as it encourages students to compare scores, and distracts their attention from how they may improve their mathematics.

Instead, help students to make further progress by asking questions that focus attention on aspects of their work. Some suggestions for these are given on the next page. These have been drawn from common difficulties anticipated.

We suggest that you write your own lists of questions, based on your own students' work, using the ideas below. You may choose to write questions on each student's work. If you do not have time to do this, select a few questions that will be of help to the majority of students. These can be written/ displayed on the board at the beginning of the lesson.

Common issues: Suggested questions and prompts:

Common Issues	Suggested questions and prompts
Student doesn't match the cards correctly because he or she doesn't have a conceptual understanding of multiplication.	<ul style="list-style-type: none"> • <i>If you are multiplying 27×4, what does the 2 represent? the 7?</i> • <i>Can you decompose the factors to make it easier to find the combined product?</i> • <i>What would happen if you multiplied 20×4 and 7×4? Could you use those answers to answer 27×4?</i>
Student doesn't understand Distributive Property. $a \times (b + c) = (a \times b) + (a \times c)$	<ul style="list-style-type: none"> • <i>How can these number(s) we are multiplying be broken apart?</i> • <i>What could you do with <u>those</u> numbers to solve this problem?</i>
Student doesn't understand the area model for multiplication.	<ul style="list-style-type: none"> • <i>In the problem 27×14 let's look at the number 27. How many 10s are in 27? How many ones? How could you model 27? Now let's look at 14? How many tens? ones? How could you model 14?</i> • <i>Is there a way to take those two models and fit them on a rectangle to discover 27×14 without doing any calculations?</i>
Student doesn't understand the lattice method for multiplication.	<ul style="list-style-type: none"> • <i>How do the numbers in the lattice grid compare to the numbers in the partial products and traditional methods?</i> • <i>How can use the numbers along each diagonal in the lattice grid to get your final product?</i>

Suggested lesson outline

Collaborative Activity: matching *Card Sets Models A, B, C, D, E and F* (30 minutes)

Organize the class into groups of two. You should only ever have one group of three (if you have an odd class size)

Give each group *Card Sets A, B, and C* – area model, lattice & Distributive Property.
Introduce the lesson carefully:

- *I want you to work as a team. Take it in turns to match a Model card with either a Lattice card or a Distributive Property card. Each time explain your thinking clearly and carefully.*
- *If you disagree with your partners' placement of a card, then respectfully challenge him/her. It is important that you both understand the math for all the placements.*

There is a lot of work to do today, and it doesn't matter if you don't all finish. The important thing is to learn something new, so take your time.

As the teacher, make a note of student approaches to the task, and support student problem solving. You can then use this information to focus a whole-class discussion towards the end of the lesson. In particular, notice any common mistakes. For example, students may know Lattice multiplication and the Distributive Property but may not understand the model.

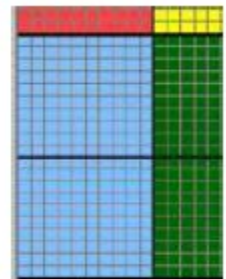
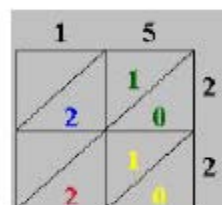
Make a note of student approaches to the task

Try not to make suggestions that move students towards a particular approach to this task. Instead, ask questions to help students clarify their thinking. Encourage students to use each other as a resource for learning.

Students will correct their own errors once the Partial Product and Traditional Algorithm cards are added.

For students struggling to get started:

- *There is more than one way to tackle this task. Can you think what one of them might be? What is it that you already know?*



$$(20 + 2) \times (10 + 5) =$$
$$20 \times 10 + 2 \times 10 + 20 \times 5 + 2 \times 5 =$$

[Working out the answer in either the Distributive Property problem or the Lattice multiplication model and then matching that card to the model or finding the answer from the model and matching it to either the Lattice or Distribution Property card.]

- *How can you calculate products with the Distributive Property? with the Lattice model?*

Example Look at this Distributive Property Card shows $(20 + 2) \times (10 + 5)$?

What would the original multiplication problem be for this model? Does that multiplication problem match any of the other cards on the table?

If one student has placed a particular card, challenge their partner to provide an explanation.

Maria placed this Lattice card with this Model. Martin, why has Maria placed it here?

If you find students have difficulty articulating their decisions, then you may want to use the questions from the *Common Issues* table to support your questioning.

If the whole class is struggling on the same issue, then you may want to write a couple of questions on the board and organize a whole class discussion.

Placing Card Sets D, E & F: Partial Products, Traditional Algorithm, & Word Problems

As students finish placing the Model, Distributive Property, and Lattice cards, hand out *Card Sets D, E & F: Partial Products, Traditional Algorithm & Word Problems*. These provide students with different ways of interpreting the situation.

Do not collect the card sets they have been using. An important part of this task is for students to make connections between all the different representations of multiplication problems.

As you monitor the work, listen to the discussion and help students to look for patterns and generalizations. Groups should have 8 different clusters of cards with 6 cards in each. The original cards show the correct matches on each row of the table as they are originally arranged.

Sharing work (10 minutes)

When students get as far as they can with matching cards, ask one student from each group to visit another group's work. Students remaining at their desk should explain their reasoning for the matched cards on their own desk.

If you are staying at your desk, be ready to explain the reasons for your group's matches.

If you are visiting another group, make note of your card placements on a piece of paper. Go to another group's desk and check to see which matches are different from your own.

If there are differences, ask for an explanation. If you still don't agree, explain your own thinking.

When you return to your own desk, you need to consider, as a group, whether to make any changes to your work.

Students may now want to make changes. Improve individual solutions to the assessment task (10 minutes) Return their original assessment, as well as a second blank copy of the task.

Look at your original responses and think about what you have learned this lesson.

Using what you have learned, try to improve your work.

If you have not added questions to individual pieces of work then write your list of questions on the board. Students should select from this list only the questions appropriate to their own work. If you find you are running out of time, then you could set this task in the next lesson, or for homework.

Solutions

Pre-Assessment: *Representing Multi-Digit Multiplication Multiple Ways*

Question 1: $28 \times 17 = 476$ Students show work, but strategies may vary

Question 2: Each of the responses from Sam, Julie, Pete, Lisa, & Fred are all correct.

- Sam used the traditional method
- Julie used lattice method,
- Pete used clustering with the distributive property & some partial products,
- Lisa used a box method that shows the partial products in the area model
- Fred drew a base-10 block area model representation.

Be sure that the responses have correct interpretations of each model, but answers may vary in the way each is described.

Question 3: Student should state which person's strategy most closely matched their own work in problem one. They should then use a different strategy to solve $39 \times 14 = 546$ correctly.

Teacher notes:

- If you find that you still have students in need of additional help you may want to look at Van de Walle's resource: *Teaching Student Centered Mathematics pages 180 – 187.*
- The *Common Core State Standards* states that students do NOT have to use the formal term *distributive property*, but they expect students to understand why this property

works because that knowledge is critical to understanding multiplication (and its ties to algebraic thinking).

- “Cluster problems” is a strategy that encourages students to use the facts that they know and allow for them to decompose larger numbers in order to solve more complex computations. When asked to multiply 34×50 students recorded the following clusters: 3×50 , 10×50 , $34 \times 25 (\times 2)$, $30 \times 50 + (4 \times 50)$

Cluster Problems help students think about ways that they can decompose numbers into easier parts (strategy: work a simpler problem)

Research also analyzed sixth graders’ varied strategies for solving multiplication problems on the criteria of flexibility, accuracy, and efficiency. Results Showed that when multiplying with two double-digit numbers only twenty percent used the standard algorithm. Be aware of that when you are analyzing their pre-assessments

Solutions for Collaborative Activity (graphic organizer provided if you need more than one day to complete lesson and have multiple classes using the cards)

Area Model Card (A)	Lattice Card (B)	Distributive Property Card (C)	Partial Products Card (D)	Traditional Card (E)	Word Problem (F)
A1	B1	C7	D6	E7	F8
A2	B2	C8	D7	E6	E7
A3	B4	C3	D1	E8	F2
A4	B7	C1	D4	E2	F5
A5	B3	C6	D8	E5	F1
A6	B5	C4	D2	E1	F3
A7	B6	C5	D3	E4	F4
A8	B8	C2	D5	E3	F6

PA- MDC: Representing Multi-Digit Multiplication Multiple Ways

Student Name _____

1. Multiply 28 by 17 and show your work and explain your approach to solving this problem.

2.) Sam, Julie, Pete, Lisa, & Fred each multiplied 28 by 17. Below each method, indicate if the work is correct and then explain what method they used and if it makes sense mathematically or not.

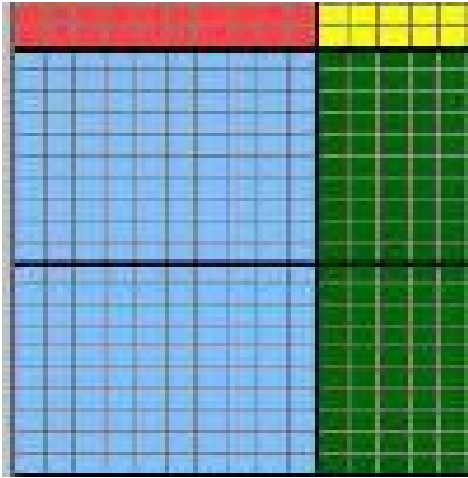
(On back of paper)

Sam	Julie	Pete	Lisa	Fred
$\begin{array}{r} 28 \\ 17 \\ \hline 196 \\ + 280 \\ \hline 476 \end{array}$		$\begin{aligned} 28 \times 10 &= 280 \\ 28 \times 5 &= 140 \\ 28 \times 2 &= 56 \\ 280 + 140 + 56 &= 476 \end{aligned}$	$\begin{array}{r} 20 \quad 8 \\ 10 \quad \begin{array}{ c c } \hline 200 & 80 \\ \hline \end{array} \\ 7 \quad \begin{array}{ c c } \hline 140 & 56 \\ \hline \end{array} \\ \hline 200 + 80 + 140 + 56 = 476 \end{array}$	
Check one: <input type="checkbox"/> correct <input type="checkbox"/> incorrect Explain why:	Check one: <input type="checkbox"/> correct <input type="checkbox"/> incorrect Explain why:	Check one: <input type="checkbox"/> correct <input type="checkbox"/> incorrect Explain why:	Check one: <input type="checkbox"/> correct <input type="checkbox"/> incorrect Explain why:	Check one: <input type="checkbox"/> correct <input type="checkbox"/> incorrect Explain why:

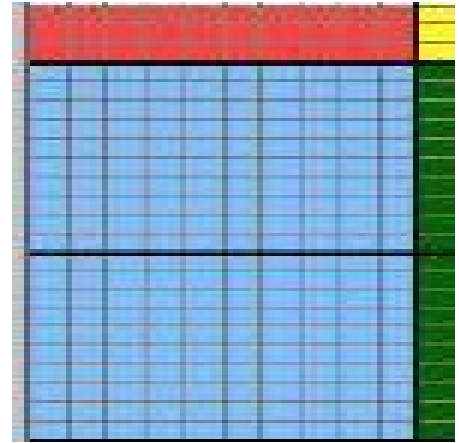
3. Which method most closely matches how you solved the original problem? _____

Choose a different method from what you used in #1 to multiply 39 by 14. Show your work.

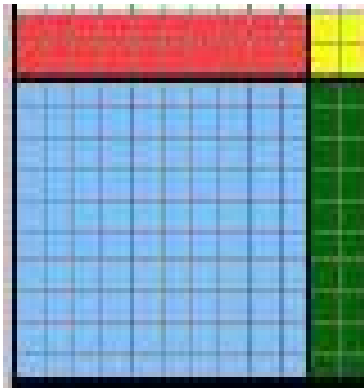
A 1



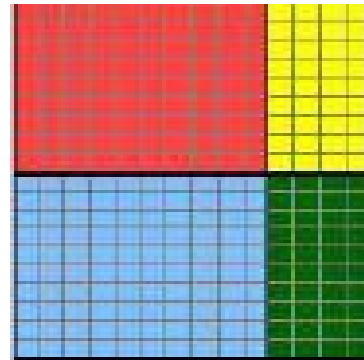
A 5



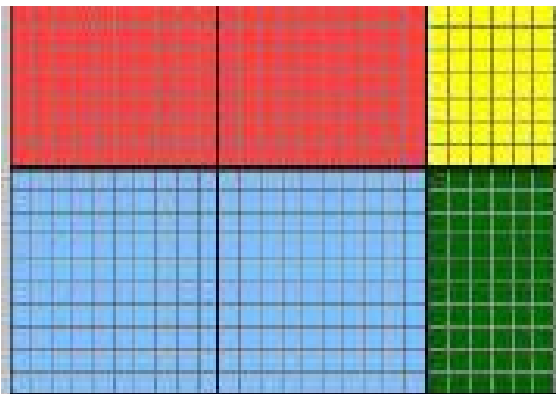
A 2



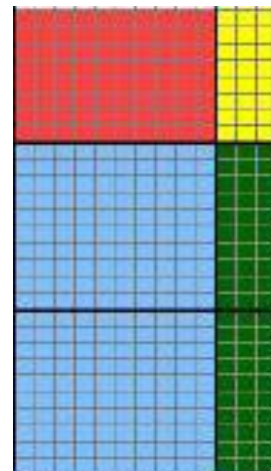
A6



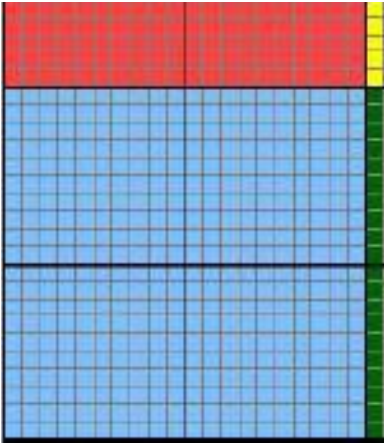
A3



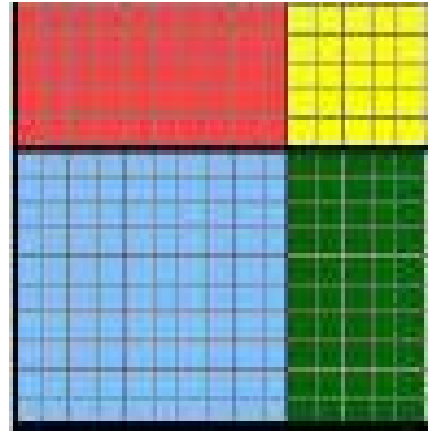
A7



A4

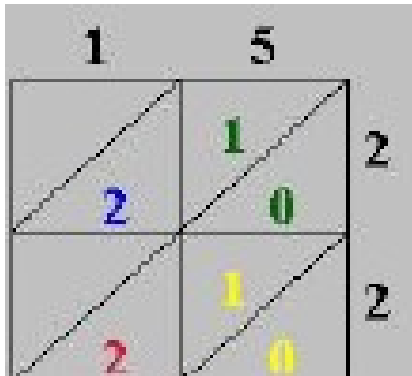


A8

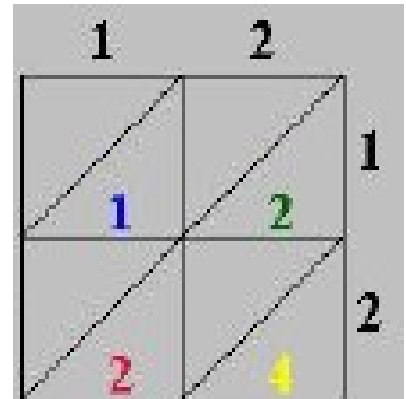


Lattice Card Set B

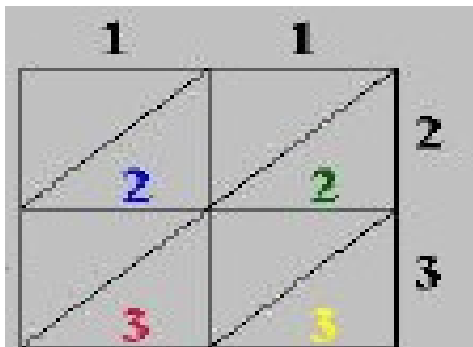
B1



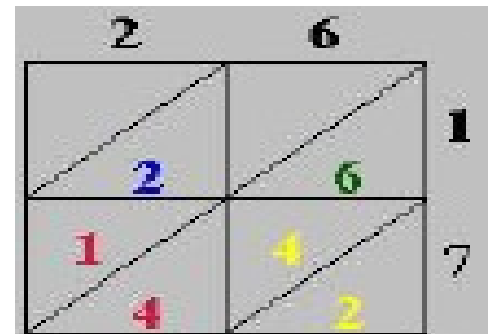
B2



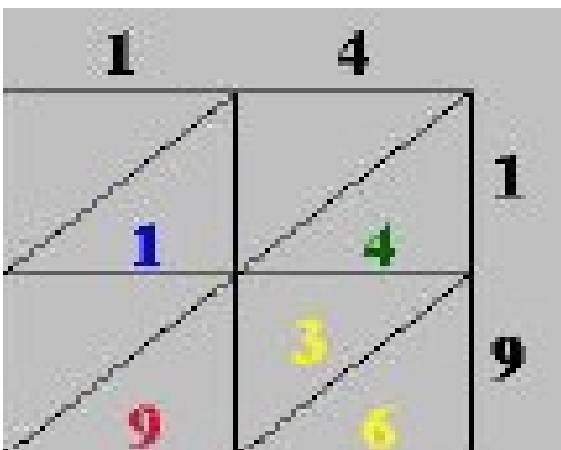
B3



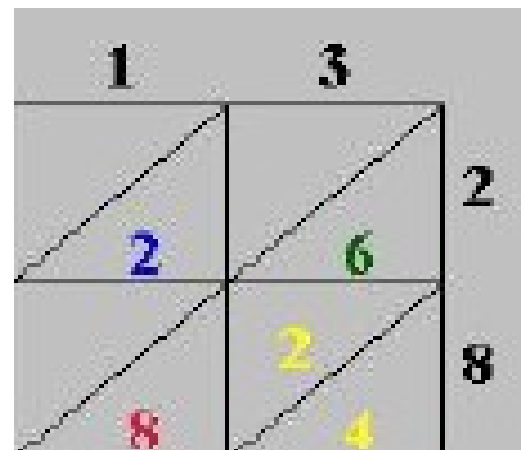
B4



B5



B6



B7

2	1	
		2
4	2	
1		5
0	5	

B8

1	5	
		1
1	5	
	2	5
5	5	

Set C Distributive Property

C1 $(20 + 5) \times (20 + 1) =$ $20 \times 20 + 5 \times 20 + 20 \times 1 + 5 \times 1 =$	C2 $(10 + 5) \times (10 + 5) =$ $10 \times 10 + 5 \times 10 + 10 \times 5 + 5 \times 5 =$
C3 $(10 + 7) \times (20 + 6) =$ $10 \times 20 + 7 \times 20 + 10 \times 6 + 7 \times 6 =$	C4 $(10 + 9) \times (10 + 4) =$ $10 \times 10 + 9 \times 10 + 10 \times 4 + 9 \times 4 =$
C5 $(20 + 8) \times (10 + 3) =$ $20 \times 10 + 8 \times 10 + 20 \times 3 + 8 \times 3 =$	C6 $(20 + 3) \times (10 + 1) =$ $20 \times 10 + 3 \times 10 + 20 \times 1 + 3 \times 1 =$
C7 $(20 + 2) \times (10 + 5) =$ $20 \times 10 + 2 \times 10 + 20 \times 5 + 2 \times 5 =$	C8 $(10 + 2) \times (10 + 2) =$ $10 \times 10 + 2 \times 10 + 10 \times 2 + 2 \times 2 =$

Card Set D Partial Products

<p>D1</p> $ \begin{array}{r} 26 \\ \times 17 \\ \hline 42 \\ 140 \\ 60 \\ \hline 200 \end{array} $	<p>D2</p> $ \begin{array}{r} 14 \\ \times 19 \\ \hline 36 \\ 90 \\ 40 \\ \hline 100 \end{array} $
<p>D 3</p> $ \begin{array}{r} 13 \\ \times 28 \\ \hline 24 \\ 80 \\ 60 \\ \hline 200 \end{array} $	<p>D4</p> $ \begin{array}{r} 21 \\ \times 25 \\ \hline 5 \\ 100 \\ 20 \\ \hline 400 \end{array} $
<p>D5</p> $ \begin{array}{r} 15 \\ \times 15 \\ \hline 25 \\ 50 \\ 50 \\ \hline 100 \end{array} $	<p>D6</p> $ \begin{array}{r} 15 \\ \times 22 \\ \hline 10 \\ 20 \\ 100 \\ \hline 200 \end{array} $

D7

12
X 12
4
20
20
100

D8

11
X 23
3
30
20
200

Card Set E Traditional Set

E 1 $\begin{array}{r} 14 \\ \times 19 \\ \hline 126 \\ 140 \\ \hline \end{array}$	E2 $\begin{array}{r} 21 \\ \times 25 \\ \hline 105 \\ 420 \\ \hline \end{array}$
E3 $\begin{array}{r} 15 \\ \times 15 \\ \hline 75 \\ 150 \\ \hline \end{array}$	E4 $\begin{array}{r} 13 \\ \times 28 \\ \hline 104 \\ 260 \\ \hline \end{array}$
E5 $\begin{array}{r} 11 \\ \times 23 \\ \hline 33 \\ 220 \\ \hline \end{array}$	E6 $\begin{array}{r} 12 \\ \times 12 \\ \hline 24 \\ 120 \\ \hline \end{array}$

E7 <div> 15 <u>X 22</u> 30 <u>300</u> </div>	E8 <div> 26 <u>X 17</u> 182 <u>260</u> </div>
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Problem Card Set F

F1 While on their campout, the boy scouts each eat twenty-three grapes. How many total grapes did the troop of eleven boys eat?	F2 The deck Scott is building needs twenty-six boards and each board needs seventeen nails. How many nails does Scott need to buy?
F3 An opossum sleeps an average of nineteen hours per day. How many hours does it sleep in a two week time period?	F4 Gloria bought thirteen different colored folders. Each folder was decorated with twenty-eight dots. How many total dots are on her folders?
F5 Each bag of Hershey kisses contains twenty-one individually wrapped kisses. How many candies are in twenty-five bags?	F6 The zoo has fifteen monkeys who eat fifteen bananas each day. How many bananas does the zoo need each day for the monkeys?
F7 How many eggs are in twelve dozen?	F8 Each pack of baseball cards has fifteen cards. How many cards are in twenty-two packs?

PA- MDC Grade 4 Representing Multi-Digit Multiplication Multiple Ways

Student Names

Area Model Card (A)	Lattice Card (B)	Distributive Property Card (C)	Partial Products Card (D)	Traditional Card (E)	Word Problem (F)
A1					
A2					
A3					
A4					
A5					
A6					
A7					
A8					

